Design Guide for Floor Vibrations
This design guide presents a method for assessing floor vibrations guaranteeing the comfort of occupants. This document is based on recent research developments (RFCS-Project “Vibration of floors”).

Guidance for the structural integrity, given in this document, is based on common approaches.
<table>
<thead>
<tr>
<th>Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>3</td>
</tr>
<tr>
<td>2. Definitions</td>
<td>7</td>
</tr>
<tr>
<td>3. Determination of Floor Characteristics</td>
<td>11</td>
</tr>
<tr>
<td>4. Classification of Vibrations</td>
<td>15</td>
</tr>
<tr>
<td>5. Design Procedure and Diagrams</td>
<td>19</td>
</tr>
<tr>
<td>Annex A Formulas for Manual Calculation</td>
<td>31</td>
</tr>
<tr>
<td>Annex B Examples</td>
<td>43</td>
</tr>
<tr>
<td>Technical assistance &amp; Finishing</td>
<td>52</td>
</tr>
<tr>
<td>References</td>
<td>52</td>
</tr>
<tr>
<td>Your partners</td>
<td>53</td>
</tr>
</tbody>
</table>
1. INTRODUCTION
Floor structures are designed for ultimate limit state and serviceability limit state criteria:

- Ultimate limit states are those related to strength and stability;
- Serviceability limit states are mainly related to vibrations and hence are governed by stiffness, masses, damping and the excitation mechanisms.

For slender floor structures, as made in steel or composite construction, serviceability criteria govern the design.

Guidance is given for:

- Specification of tolerable vibration by the introduction of acceptance classes (Chapter 4) and
- Prediction of floor response due to human induced vibration with respect to the intended use of the building (Chapter 5).

An overview of the general design procedure presented in Chapter 5 is given in Figure 1.

For the prediction of vibration, several dynamic floor characteristics need to be determined. These characteristics and simplified methods for their determination are briefly described. Design examples are given in Annex B of this design guide.

The design guide comprehends simple methods, design tools and recommendations for the acceptance of vibration of floors which are caused by people during normal use. The given design methods focus on the prediction of vibration. Measurements performed after erection may lead to differences to the predicted values so that one cannot claim on the predicted result.

The design and assessment methods for floor vibrations are related to human induced resonant vibrations, mainly caused by walking under normal conditions. Machine induced vibrations or vibrations due to traffic etc. are not covered by this design guide.

The design guide should not be applied to pedestrian bridges or other structures, which do not have a structural characteristic or a characteristic of use comparable to floors in buildings.
1. Introduction

Figure 1  Design procedure (see Chapter 5)

Determine dynamic floor characteristics:

- Natural Frequency
- Modal Mass
- Damping

(Chapter 3; Annex A)

Read off OS-RMS$_{90}$ – Value
(Chapter 5)

Determine Acceptance Class
(Chapter 4)
2. DEFINITIONS
2. Definitions

The definitions given here are consistent with the application of this design guide.

**Damping** $D$

Damping is the energy dissipation of a vibrating system. The total damping consists of:

- Material and structural damping,
- Damping by furniture and finishing (e.g. false floor),
- Spread of energy throughout the whole structure.

**Modal mass** $M_{mod} = \text{generalised mass}$

Each mode of a system with several degrees of freedom can be represented by a system with a single degree of freedom:

$$f = \frac{1}{2\pi} \sqrt{\frac{K_{mod}}{M_{mod}}}$$

where

- $f$ is the natural frequency of the considered mode,
- $K_{mod}$ is the modal stiffness,
- $M_{mod}$ is the modal mass.

Thus the modal mass can be interpreted to be the mass activated in a specific mode shape. The determination of the modal mass is described in Chapter 3.
2. Definitions

**Natural Frequency** \( f = \) Eigenfrequency

Every structure has its specific dynamic behaviour with regard to shape and duration \( T \) [s] of a single oscillation. The frequency \( f \) is the reciprocal of the oscillation time \( T(f = 1/T) \).

The natural frequency is the frequency of a free oscillation without continuously being driven by an exciter.

Each structure has as many natural frequencies and associated mode shapes as degrees of freedom. They are commonly sorted by the amount of energy that is activated by the oscillation. Therefore the first natural frequency is that on the lowest energy level and is thus the most likely to be activated.

The equation for the natural frequency of a single degree of freedom system is:

\[
f = \frac{1}{2\pi} \sqrt{\frac{K}{M}}
\]

where \( \frac{K}{M} \) is the stiffness, \( M \) is the mass.

The determination of frequencies is described in Chapter 3.

---

**OS-RMS\(_{90} = \)**

RMS - value of the velocity for a significant step covering the intensity of 90% of people’s steps walking normally.

- OS: One step
- \( v_{\text{RMS}} \): Root mean square = effective value, here of velocity \( v \).

\[
v_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 \, dt} = \frac{v_{\text{Peak}}}{\sqrt{2}}
\]

where \( T \) is the investigated period of time.
3. DETERMINATION OF FLOOR CHARACTERISTICS
3. Determination of Floor Characteristics

The determination of floor characteristics can be performed by simple calculation methods, by Finite Element Analysis (FEA) or by testing. As the design guide is intended to be used for the design of new buildings, testing procedures are excluded from further explanations and reference is given to [1].

Different finite element programs can perform dynamic calculations and offer tools for the determination of natural frequencies. The model mass is in many programs also a result of the analysis of the frequency. As it is specific for each software what elements can be used, how damping is considered and how and which results are given by the different programs, only some general information is given in this design guide concerning FEA.

If FEA is applied for the design of a floor with respect to the vibration behaviour, it should be considered that the FE-model for this purpose may differ significantly to that used for ultimate limit state (ULS) design as only small deflections are expected due to vibration. A typical example is the different consideration of boundary conditions in vibration analysis. If compared to ULS design: a connection which is assumed to be a hinged connection in ULS may be rather assumed to provide full moment connection in vibration analysis.

For concrete, the dynamic modulus of elasticity should be considered to be 10% higher than the static modulus $E_{cm}$.

For manual calculations, Annex A gives formulas for the determination of frequency and modal mass for isotropic plates, orthotropic plates and beams.

Damping has a big influence on the vibration behaviour of a floor. Independent on the method chosen to determine the natural frequency and modal mass the damping values for a vibrating system can be determined with the values given in Table 1. These values are considering the influence of structural damping for different materials, damping due to furniture and damping due to finishes. The system damping $D$ is obtained by summing up the appropriate values for $D_1$ to $D_3$.

In the determination of the dynamic floor characteristics, a realistic fraction of imposed load should be considered in the mass of the floor $(m, M)$. Experienced values for residential and office buildings are 10% to 20% of the imposed load.
### Table 1: Determination of damping

<table>
<thead>
<tr>
<th>Type</th>
<th>Damping (% of critical damping)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural Damping $D_1$</td>
<td></td>
</tr>
<tr>
<td>Wood</td>
<td>6%</td>
</tr>
<tr>
<td>Concrete</td>
<td>2%</td>
</tr>
<tr>
<td>Steel</td>
<td>1%</td>
</tr>
<tr>
<td>Composite (steel-concrete)</td>
<td>1%</td>
</tr>
<tr>
<td>Damping due to furniture $D_2$</td>
<td></td>
</tr>
<tr>
<td>Traditional office for 1 to 3 persons with separation walls</td>
<td>2%</td>
</tr>
<tr>
<td>Paperless office</td>
<td>0%</td>
</tr>
<tr>
<td>Open plan office</td>
<td>1%</td>
</tr>
<tr>
<td>Library</td>
<td>1%</td>
</tr>
<tr>
<td>Houses</td>
<td>1%</td>
</tr>
<tr>
<td>Schools</td>
<td>0%</td>
</tr>
<tr>
<td>Gymnastic</td>
<td>0%</td>
</tr>
<tr>
<td>Damping due to finishes $D_3$</td>
<td></td>
</tr>
<tr>
<td>Ceiling under the floor</td>
<td>1%</td>
</tr>
<tr>
<td>Free floating floor</td>
<td>0%</td>
</tr>
<tr>
<td>Swimming screed</td>
<td>1%</td>
</tr>
</tbody>
</table>

Total Damping $D = D_1 + D_2 + D_3$
4. CLASSIFICATION OF VIBRATIONS
4. Classification of Vibrations

The perception of vibrations by persons and the individual feeling of annoyance depends on several aspects. The most important are:

- The direction of the vibration, however in this design guide only vertical vibrations are considered;
- Another aspect is the posture of people such as standing, lying or sitting;
- The current activity of the person considered is of relevance for its perception of vibrations. Persons working in the production of a factory perceive vibrations differently from those working concentrated in an office or a surgery;
- Additionally, age and health of affected people may be of importance for feeling annoyed by vibrations.

Thus the perception of vibrations is a very individual problem that can only be described in a way that fulfils the acceptance of comfort of the majority.

It should be noted that the vibrations considered in this design guide are relevant for the comfort of the occupants only. They are not relevant for the structural integrity.

Aiming at an universal assessment procedure for human induced vibration, it is recommended to adopt the so-called one step RMS value (OS-RMS) as a measure for assessing annoying floor vibrations. The OS-RMS values correspond to the harmonic vibration caused by one relevant step onto the floor.

As the dynamic effect of people walking on a floor depends on several boundary conditions, such as weight and speed of walking of the people, their shoes, flooring, etc., the 90% OS-RMS (OS-RMS$_{90}$) value is recommended as assessment value. The index 90 indicates that 90 percent of steps on the floor are covered by this value.

The following table classifies vibrations into several classes and gives also recommendations for the assignment of classes with respect to the function of the considered floor.
### Table 2 Classification of floor response and recommendation for the application of classes

<table>
<thead>
<tr>
<th>OS-RMS&lt;sub&gt;90&lt;/sub&gt;</th>
<th>Function of Floor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Class</strong></td>
<td>Lower Limit</td>
</tr>
<tr>
<td>A</td>
<td>0.0</td>
</tr>
<tr>
<td>B</td>
<td>0.1</td>
</tr>
<tr>
<td>C</td>
<td>0.2</td>
</tr>
<tr>
<td>D</td>
<td>0.8</td>
</tr>
<tr>
<td>E</td>
<td>3.2</td>
</tr>
<tr>
<td>F</td>
<td>12.8</td>
</tr>
</tbody>
</table>

Legend:
- Green: Recommended
- Orange: Critical
- Red: Not recommended

4. Classification of vibrations
5. DESIGN PROCEDURE AND DIAGRAMS
5. Design Procedure and Diagrams

An overview of the general design procedure is given in Figure 2. The design is carried out in 3 steps where the determination of the dynamic floor characteristics is the most complex one. Thus Annex A gives detailed help by simplified methods; general explanations are given in Chapter 3.

When modal mass and frequency are determined, the OS–RMS$_{90}$–value as well as the assignment to the perception classes may be determined with the diagrams given below. The relevant diagram needs to be selected according to the damping characteristics of the floor in the condition of use (considering finishing and furniture), see Chapter 3.

The diagrams have been elaborated by TNO Bouw, the Netherlands, in the frame of [1].

**Figure 2** Design procedure

**Determine dynamic floor characteristics:**
- Natural Frequency
- Modal Mass
- Damping
  (Chapter 3; Annex A)

**Read off OS–RMS$_{90}$ – Value**
  (Chapter 5)

**Determine Acceptance Class**
  (Chapter 4)
Figure 3  Application of diagrams

The diagram is used by entering the modal mass on the x-axis and the corresponding frequency on the y-axis. The OS-RMS value and the acceptance class can be read-off at the intersection of extensions at both entry points.

: means out of the range of a tolerable assessment
Classification based on a damping ratio of 1%
Figure 5  OS-RMS\textsubscript{90} for 2\% Damping

Classification based on a damping ratio of 2\%
Classification based on a damping ratio of 3%
5. Design Procedure and Diagrams

Figure 7  OS-RMS\textsubscript{90} for 4\% Damping

Classification based on a damping ratio of 4\%
Figure 8  OS-RMS\(_{90}\) for 5% Damping

Classification based on a damping ratio of 5%
5. Design Procedure and Diagrams

Figure 9  OS-RMS\textsubscript{60} for 6% Damping

Classification based on a damping ratio of 6%
5. Design Procedure and Diagrams

Figure 10  OS-RMS\textsubscript{90} for 7\% Damping

Classification based on a damping ratio of 7\%

![Diagram showing modal mass and eigenfrequency classification]

Modal mass of the floor [kg]

Eigenfrequency of the floor [Hz]
Figure 11  OS-RMS$_{90}$ for 8% Damping

Classification based on a damping ratio of 8%

Modal mass of the floor [kg]

Eigenfrequency of the floor [Hz]
5. Design Procedure and Diagrams

Figure 12  OS-RMS\textsubscript{90} for 9\% Damping

Classification based on a damping ratio of 9\%

![Diagram of OS-RMS\textsubscript{90} for 9\% Damping]

- Eigenfrequency of the floor (Hz)
- Modal mass of the floor (kg)
## ANNEX A  FORMULAS FOR MANUAL CALCULATION

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>Natural Frequency and Modal Mass for Isotropic Plates</td>
<td>32</td>
</tr>
<tr>
<td>A.2</td>
<td>Natural Frequency and Modal Mass for Beams</td>
<td>34</td>
</tr>
<tr>
<td>A.3</td>
<td>Natural Frequency and Modal Mass for Orthotropic Plates</td>
<td>35</td>
</tr>
<tr>
<td>A.4</td>
<td>Self Weight Approach for Natural Frequency</td>
<td>36</td>
</tr>
<tr>
<td>A.5</td>
<td>Dunkerley Approach for Natural Frequency</td>
<td>37</td>
</tr>
<tr>
<td>A.6</td>
<td>Approximation of Modal Mass</td>
<td>38</td>
</tr>
</tbody>
</table>
A.1 Natural Frequency and Modal Mass for Isotropic Plates

The following table gives formulas for the determination of the first natural frequency (acc. to [2]) and the modal mass of plates for different support conditions. For the application of the given equations, it is assumed that no lateral deflection at any edges of the plate occurs.

Support Conditions: clamped hinged

\[ f = \frac{\alpha}{l^2} \sqrt{\frac{E t^3}{12 m (1-v^2)}} \]

\[ \lambda \]

\[ \beta \]

\[ \alpha = 1.57 \left( 1 + \lambda^2 \right) \]

\[ \alpha = 1.57 \sqrt{1 + 2.5 \lambda^2 + 5.14 \lambda^4} \]

\[ \alpha = 1.57 \sqrt{5.14 + 2.92 \lambda^2 + 2.44 \lambda^4} \]

\[ \beta = 0.20 \text{ for all } \lambda \]

\[ \beta = 0.18 \text{ for all } \lambda \]

\[ \text{Ratio } \lambda = l/b \]
Support Conditions:

- clamped 
- hinged

Frequency; Modal Mass

\[ f = \frac{\alpha}{l^2} \sqrt{\frac{E t^3}{12 m (1 - \nu^2)}} \quad ; \quad M_{\text{mod}} = \beta M \]

\[ \alpha = 1.57 \sqrt{1 + 2.33 \lambda^2 + 2.44 \lambda^4} \]
\[ \beta = 0.22 \text{ for all } \lambda \]

\[ \alpha = 1.57 \sqrt{2.44 + 2.72 \lambda^2 + 2.44 \lambda^4} \]
\[ \beta = 0.21 \text{ for all } \lambda \]

\[ \alpha = 1.57 \sqrt{5.14 + 3.13 \lambda^2 + 5.14 \lambda^4} \]
\[ \beta = 0.17 \text{ for all } \lambda \]
A.2 Natural Frequency and Modal Mass for Beams

The first Eigenfrequency of a beam can be determined with the formula according to the supporting conditions from Table 3 with:

\[ \begin{align*}
E & \quad \text{Young's modulus [N/m²]} \\
I & \quad \text{Moment of inertia [m}^4]\] \\
\mu & \quad \text{Distributed mass } m \text{ of the floor (see page 33) multiplied by the floor width [kg/m]} \\
l & \quad \text{Length of beam [m]}
\end{align*} \]

\[ f = \frac{4}{\pi} \sqrt{\frac{3 EI}{0.37 \mu l^4}} \]
\[ M_{mod} = 0.41 \mu l \]

\[ f = \frac{2}{\pi} \sqrt{\frac{3 EI}{0.2 \mu l^2}} \]
\[ M_{mod} = 0.45 \mu l \]

\[ f = \frac{2}{\pi} \sqrt{\frac{3 EI}{0.49 \mu l^2}} \]
\[ M_{mod} = 0.5 \mu l \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{3 EI}{0.24 \mu l^2}} \]
\[ M_{mod} = 0.64 \mu l \]

**Table 3** Determination of the first Eigenfrequency of Beams

<table>
<thead>
<tr>
<th>Support Conditions</th>
<th>Natural Frequency</th>
<th>Modal Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f = \frac{4}{\pi} \sqrt{\frac{3 EI}{0.37 \mu l^4}} )</td>
<td>( M_{mod} = 0.41 \mu l )</td>
</tr>
<tr>
<td></td>
<td>( f = \frac{2}{\pi} \sqrt{\frac{3 EI}{0.2 \mu l^2}} )</td>
<td>( M_{mod} = 0.45 \mu l )</td>
</tr>
<tr>
<td></td>
<td>( f = \frac{2}{\pi} \sqrt{\frac{3 EI}{0.49 \mu l^2}} )</td>
<td>( M_{mod} = 0.5 \mu l )</td>
</tr>
<tr>
<td></td>
<td>( f = \frac{1}{2\pi} \sqrt{\frac{3 EI}{0.24 \mu l^2}} )</td>
<td>( M_{mod} = 0.64 \mu l )</td>
</tr>
</tbody>
</table>
A.3 Natural Frequency and Modal Mass for Orthotropic Plates

Orthotropic floors as e.g. composite floors with beams in longitudinal direction and a concrete plate in transverse direction have different stiffnesses in length and width ($EI_y > EI_x$). An example is given in Figure 13.

The first natural frequency of the orthotropic plate being simply supported at all four edges can be determined with:

$$f_1 = \frac{\pi}{2} \sqrt{\frac{EI_y}{ml^2}} \sqrt{1 + \left[ 2 \left( \frac{b}{l} \right)^2 + \left( \frac{b}{l} \right)^4 \right] \frac{EI_x}{EI_y}}$$

Where:

- $m$ is the mass of floor including finishes and a representative amount of imposed load (see chapter 3) [kg/m²],
- $l$ is the length of the floor (in x-direction) [m],
- $b$ is the width of the floor (in y-direction) [m],
- $E$ is the Young’s modulus [N/m²],
- $I_x$ is the moment of inertia for bending about the x-axis [m⁴],
- $I_y$ is the moment of inertia for bending about the y-axis [m⁴].

Formulas for the approximation of the modal mass for orthotropic plates are given in Annex A.6.

Figure 13 Dimensions and axis of an orthotropic plate
The self weight approach is a very practical approximation in cases where the maximum deflection $\delta_{\text{max}}$ due to the mass $m$ is already determined, e.g. by finite element calculation.

This method has its origin in the general frequency equation:

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$$

The stiffness $K$ can be approximated by the assumption:

$$K = \frac{M \cdot g}{\frac{3}{4} \bar{\delta}}$$

where:

- $M$ is the total mass of the vibrating system [kg],
- $g = 9.81$ is the gravity [m/s$^2$] and
- $\frac{3}{4} \bar{\delta}$ is the average deflection [mm].

The approximated natural frequency is

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{M}} = \frac{1}{2\pi} \sqrt{\frac{4g}{3\delta_{\text{max}}}} = \frac{18}{\sqrt{\delta_{\text{max}}} [\text{mm}]}$$

where:

$\delta_{\text{max}}$ is the maximum deflection due to loading in reference to the mass $m$. 
The Dunkerley approach is an approximation for manual calculations. It is applied when the expected mode shape is complex but can be subdivided into different single modes for which the natural frequency can be determined, see A.1, A.3 and A.2.

Figure 14 shows an example of a composite floor with two simply supported beams and no support at the edges of the concrete plate. The expected mode shape is divided into two independent single mode shapes; one of the concrete slab and one of the composite beam. Both mode shapes have their own natural frequency ($f_1$ for the mode of the concrete slab and $f_2$ for the composite beam).

According to Dunkerley, the resulting natural frequency $f$ of the total system is:

$$\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2} + \frac{1}{f_3^2} + \ldots$$

\[\text{Figure 14} \quad \text{Example for mode shape decomposition}\]
A.6 Approximation of Modal Mass

The modal mass may be interpreted as the fraction of the total mass of a floor that is activated when the floor oscillates in a specific mode shape. Each mode shape has its specific natural frequency and modal mass.

For the determination of the modal mass the mode shape has to be determined and to be normalised to the maximum deflection. As the mode shape cannot be determined by manual calculations, approximations for the first mode are commonly used.

As an alternative to manual calculations, Finite Element Analysis is commonly used. If the Finite Element software does not give modal mass as result of modal analysis, the mode shape may be approximated by the application of loads driving the plate into the expected mode shape, see Figure 15.

Figure 15 Application of load to obtain approximated load shape (example)

Expected mode shape:

Application of loads:

If the mode shape of a floor can be approximated by a normalised function \( f(x,y) \) (i.e. \( |f(x,y)|_{\text{max}} = 1.0 \)) the corresponding modal mass of the floor can be calculated by the following equation:

\[
M_{\text{mod}} = \mu \int f(x,y)^2 \, dF
\]

Where:

- \( \mu \) is the distribution of mass
- \( f(x,y) \) is the vertical deflection at location \( x,y \)

When mode shape deflections are determined by FEA:

\[
M_{\text{mod}} = \sum_{\text{Nodes } i} f_i^2 \, dM_i
\]

Where:

- \( f_i \) is the vertical deflection at node \( i \) (normalised to the maximum deflection)
- \( dM_i \) is the mass of the floor represented at node \( i \)

If the function \( f(x,y) \) represents the exact solution for the mode shape the above described equation also yields to the exact modal mass.
The following gives examples for the determination of modal mass by manual calculation:

**Example 1**

Plate simply supported along all four edges, \( l_y \sim l_x \)

- Approximation of the first mode shape:
  \[
  f(x, y) = \sin\left(\frac{\pi x}{l_x}\right) \sin\left(\frac{\pi y}{l_y}\right)
  \]
  \[
  \left| f(x, y) \right|_{\text{max}} = 1.0
  \]

- Mass distribution
  \[
  \mu = \frac{M}{l_x l_y}
  \]

- Modal mass
  \[
  M_{\text{mod}} = \mu \int f^2(x, y) \, dF = \frac{M}{l_x l_y} \int_0^{l_x} \int_0^{l_y} \sin^2\left(\frac{\pi x}{l_x}\right) \sin^2\left(\frac{\pi y}{l_y}\right) \, dx \, dy
  \]
  \[
  = \frac{M}{4}
  \]
Example 2

Plate simply supported along all four edges, \( l_y \ll l_x \).

- Approximation of the first mode shape:

1. \( 0 \leq y \leq \frac{l_y}{2} \) and \( l_y - \frac{l_x}{2} \leq y \leq l_y \) : \( f(x, y) = \sin \left( \frac{\pi x}{l_x} \right) \sin \left( \frac{\pi y}{l_y} \right) \)

\[ |f(x, y)|_{\max} = 1.0 \]

2. \( \frac{l_x}{2} \leq y \leq l_y - \frac{l_y}{2} \) : \( f(x, y) = \sin \left( \frac{\pi x}{l_x} \right) \) 1.0

\[ |f(x, y)|_{\max} = 1.0 \]

- Mass distribution

\[ \mu = \frac{M}{l_x l_y} \]

- Modal mass

\[ M_{\text{mod}} = \mu \int_{F} f^2(x, y) \, dF \]

\[ = \frac{M}{l_x l_y} \left[ 2 \int_{0}^{l_y} \int_{0}^{l_x} \sin^2 \left( \frac{\pi x}{l_x} \right) \sin^2 \left( \frac{\pi y}{l_y} \right) \, dx \, dy \right. \]

\[ + \int_{0}^{l_y} \int_{y = l_y - \frac{2l_x}{l_y}}^{l_y} \frac{1}{2} \sin^2 \left( \frac{\pi x}{l_x} \right) \, dx \, dy \]

\[ = \frac{M}{4} \left( 2 - \frac{l_x}{l_y} \right) \]
Example 3

Plate spans uniaxial between beams, plate and beams simply supported

- Approximation of the first mode shape:

\[
f(x, y) = \frac{\delta_x}{\delta} \sin\left(\frac{\pi x}{l_x}\right) + \frac{\delta_y}{\delta} \sin\left(\frac{\pi y}{l_y}\right)
\]

\[
|f(x, y)|_{\text{max}} = 1.0
\]

With

\[
\delta_x = \text{Deflection of the beam}
\]

\[
\delta_y = \text{Deflection of the slab assuming the deflection of the supports (i.e. the deflection of the beam) is zero}
\]

\[
\delta = \delta_x + \delta_y
\]

- Mass distribution

\[
\mu = \frac{M}{l_x l_y}
\]

- Modal mass

\[
M_{\text{mod}} = \mu \int \int f^2(x, y) \, dF = \frac{M}{l_x l_y} \int_0^{l_y} \int_0^{l_x} \left[ \frac{\delta_x}{\delta} \sin\left(\frac{\pi x}{l_x}\right) + \frac{\delta_y}{\delta} \sin\left(\frac{\pi y}{l_y}\right) \right]^2 \, dx \, dy
\]

\[
= M \left[ \frac{\delta_x^2 + \delta_y^2}{2\delta^2} + \frac{8}{\pi^2} \frac{\delta_x \delta_y}{\delta^2} \right]
\]
ANNEX B  EXAMPLES

B.1  Filigree slab with ACB-composite beams (office building)  44
B.1.1  Description of the Floor  44
B.1.2  Determination of dynamic floor characteristics  47
B.1.3  Assessment  47
B.2  Three storey office building  48
B.2.1  Description of the Floor  48
B.2.2  Determination of dynamic floor characteristics  50
B.2.3  Assessment  51
B.1 Filigree slab with ACB-composite beams (office building)

B.1.1 Description of the Floor

In the first worked example, a filigree slab with false-floor in an open plan office is checked for footfall induced vibrations.

The slab spans uniaxially by over 4.2 m between main beams. Its overall thickness is 160 mm. The main beams are ArcelorMittal Cellular Beams (ACB) which act as composite beams. They are attached to the vertical columns by a full moment connection. The floor plan is shown in Figure 18. For a vibration analysis it is sufficient to check only a part of the floor (representative floor bay). The representative part of the floor to be considered in this example is indicated by the hatched area in Figure 18.

Figure 16 Building structure

Figure 17 Beam to column connection

Figure 18 Floor plan
For the main beams (span of 16.8 m) ACB/HEM400/HEB400 profiles in steel S460 have been used, see Figure 19. The main beams with the shorter span of 4.2 m are ACB/HEM360 in S460.

The cross beams which are spanning in global x-direction may be neglected for the further calculations, as they do not contribute to the load transfer of the structure.

The nominal material properties are

- Steel S460: $E_s = 210000 \text{ N/mm}^2$, $f_y = 460 \text{ N/mm}^2$
- Concrete C25/30: $E_{cm} = 31000 \text{ N/mm}^2$, $f_{ck} = 25 \text{ N/mm}^2$

As stated in Chapter 3, the nominal elastic modulus of the concrete will be increased for the dynamic calculations:

$$E_{c,dyn} = 1.1 \times E_{cm} = 34100 \text{ N/mm}^2$$

The expected mode shape of the considered part of the floor which corresponds to the first Eigenfrequency is shown in Figure 20. From the mode shape it can be concluded that each field of the concrete slab may be assumed to be simply supported for the further dynamic calculations.

Regarding the boundary conditions of the main beams (see beam to column connection in Figure 17) it is assumed that for small amplitudes as they occur in vibration analysis, the beam-column connection provides sufficient rotational restrain, i.e. the main beams are considered to be fully fixed.
Section properties

Slab
The relevant section properties of the slab in global x-direction are:

\[ A_{c,x} = 160 \text{ mm}^2/\text{mm} \]
\[ I_{c,x} = 3.41 \times 10^5 \text{ mm}^4/\text{mm} \]

Main beam
Assuming the previously described first vibration mode, the effective width of the composite beam may be obtained from the following equation:

\[ b_{\text{eff}} = b_{\text{eff,1}} + b_{\text{eff,2}} = \frac{l_0}{8} + \frac{l_0}{8} \]
\[ = 2 \frac{0.7 \times 16.8}{8} = 2.94 \text{ m} \]

The relevant section properties of the main beam for serviceability limit state (no cracking) are:

\[ A_{a,\text{net}} = 21936 \text{ mm}^2 \]
\[ A_{a,\text{total}} = 29214 \text{ mm}^2 \]
\[ A_i = 98320 \text{ mm}^2 \]
\[ I_i = 5.149 \times 10^9 \text{ mm}^4 \]

Loads

Slab
- Self weight (includes 1.0 kN/m² for false floor):

\[ g_{\text{slab}} = 160 \times 10^{-3} \times 25 + 1.0 = 5 \text{ kN/m}^2 \]

- Live load: Usually a characteristic live load of 3 kN/m² is recommended for floors in office buildings. The considered fraction of the live load for the dynamic calculation is assumed to be approx. 10% of the full live load, i.e. for the vibration check it is assumed that

\[ q_{\text{slab}} = 0.1 \times 3.0 = 0.3 \text{ kN/m}^2 \]

Main beam
- Self weight (includes 2.00 kN/m for ACB):

\[ g_{\text{beam}} = 5.0 \times \frac{4.2}{2} \times 2 + 2.0 = 23.00 \text{ kN/m}^2 \]

- Live load:

\[ q_{\text{slab}} = 0.3 \times \frac{4.2}{2} \times 2 = 1.26 \text{ kN/m} \]
B.1.2 Determination of dynamic floor characteristics

**Eigenfrequency**

The first Eigenfrequency is calculated based on the self weight approach. The maximum total deflection may be obtained by superposition of the deflection of the slab and the deflection of the main beam, i.e.

\[ \delta_{max} = \delta_{slab} + \delta_{beam} \]

With

\[ \delta_{slab} = \frac{5 \times (5.0 + 0.3) \times 10^{-3} \times 4200^4}{384 \times 34100 \times 3.41 \times 10^5} = 1.9 \text{ mm} \]

\[ \delta_{beam} = \frac{1 \times (23.0 + 1.26) \times 16800^4}{384 \times 210000 \times 5.149 \times 10^9} = 4.7 \text{ mm} \]

Thus the maximum deflection is

\[ \delta_{max} = 1.9 + 4.7 = 6.6 \text{ mm} \]

Thus the first Eigenfrequency may be obtained (according to Annex A.4) from

\[ f_1 = \frac{18}{\sqrt{6.6}} = 7.0 \text{ Hz} \]

**Modal Mass**

The total mass of the considered floor bay is

\[ M = (5 + 0.3) \times 10^2 \times 16.8 \times 4.2 = 37397 \text{ kg} \]

According to Chapter A.6, Example 3, the modal mass of the considered slab may be calculated as

\[ M_{mod} = 37397 \times \left[ \frac{1.9^2 + 4.5^2}{2 \times 6.4^2} + \frac{8}{\pi^2} \times \frac{1.9 \times 4.5}{6.4^2} \right] = 17220 \text{ kg} \]

B.1.3 Assessment

Based on the above calculated modal properties the floor is classified as class C (Figure 6). The expected OS-RMS value is approx. 0.5 mm/s.

According to Table 2, class C is classified as being suitable for office buildings, i.e. the requirements are fulfilled.
B.2 Three storey office building

B.2.1 Introduction

The method leads in general to conservative results when applied as single bay method using the mode related to the fundamental frequency. However, in special cases in which the modal mass for a higher mode is significantly low, also higher modes need to be considered, see the following example.

B.2.2 Description of the Floor

The floor of this office building, Figure 21, spans 15 m from edge beam to edge beam. In the regular area these secondary floor beams are IPE600 sections, spaced in 2.5 m. Primary edge beams, which span 7.5 m from column to column, consist also of IPE600 sections, see Figure 22.

Figure 21 Building overview

Figure 22 Plan view of floor with choice of sections
The slab is a composite plate of 15 cm total thickness with steel sheets COFRASTRA 70, Figure 23. More information concerning the COFRASTRA 70 are available on www.arval-construction.fr. The nominal material properties are:

- Steel S235: \( E_s = 210000 \text{ N/mm}^2, \quad f_y = 235 \text{ N/mm}^2 \)
- Concrete C25/30: \( E_{cm} = 31000 \text{ N/mm}^2, \quad f_{ck} = 25 \text{ N/mm}^2 \)

For dynamic calculations (vibration analysis) the elastic modulus will be increased according to Chapter 3.

\[ E_{c, \text{dyn}} = 1.1 \times E_{cm} = 34100 \text{ N/mm}^2 \]

**Section properties**

**Slab (transversal to beam, \( E = 210000 \text{ N/mm}^2 \))**

\[
\begin{align*}
A & = 1170 \text{ cm}^2/\text{m} \\
I & = 20355 \text{ cm}^4/\text{m}
\end{align*}
\]

**Composite beam** (\( b_{\text{eff}} = 2.5 \text{ m}; E = 210000 \text{ N/mm}^2 \))

\[
\begin{align*}
A & = 468 \text{ cm}^2 \\
I & = 270089 \text{ cm}^4
\end{align*}
\]

**Loads**

**Slab**

Self weight:

\[
\begin{align*}
g & = 3.5 \text{ kN/m}^2 \\
\Delta g & = 0.5 \text{ kN/m}^2 \\
g + \Delta g & = 4.0 \text{ kN/m}^2 \text{ (permanent load)}
\end{align*}
\]

Life load:

\[
\begin{align*}
q & = 3.0 \times 0.1 = 0.3 \text{ kN/m}^2 \\
& \quad \text{(10% of full live load)}
\end{align*}
\]

**Composite beam**

Self weight:

\[
\begin{align*}
g & = (3.5+0.5) \times 2.5 + 1.22 = 11.22 \text{ kN/m}
\end{align*}
\]

Life load:

\[
\begin{align*}
q & = 0.3 \times 2.5 = 0.75 \text{ kN/m}
\end{align*}
\]
B.2.3 Determination of dynamic floor characteristics

Supporting conditions

The secondary beams are ending in the primary beams which are open sections with low torsional stiffness. Thus these beams may be assumed to be simply supported.

Eigenfrequency

In this example, the Eigenfrequency is determined according to three methods: the beam formula, neglecting the transversal stiffness of the floor, the formula for orthotropic plates and the self-weight method considering the transversal stiffness.

- Application of the beam equation (Chapter A.2):

\[ p = 11.97 \text{ kN/m} \Rightarrow \mu = 11.97 \times 1000 / 9.81 = 1220 \text{ kg/m} \]

\[ f = \frac{2}{\pi} \sqrt{\frac{3}{0.49 \mu^2}} = \frac{2}{\pi} \sqrt{\frac{3 \times 210000 \times 10^6 \times 270089 \times 10^{-8}}{0.49 \times 1220 \times 15^4}} = 4.8 \text{Hz} \]

- Application of the equation for orthotropic plates (Chapter A.3):

\[ f_i = \frac{\pi}{2} \sqrt{\frac{EI_y}{m l^4}} \left[ 1 + \left\{ \left( \frac{b}{l} \right)^2 \right\} \frac{EI_x}{EI_y} \right] \]

\[ = \frac{\pi}{2} \sqrt{\frac{210000 \times 10^6 \times 270089 \times 10^{-8}}{1220 \times 15^4}} \times \sqrt{1 + \left[ \left( \frac{2.5}{15} \right)^2 + \left( \frac{2.5}{15} \right)^4 \right] \frac{3410 \times 20355}{21000 \times 270089}} \]

\[ = 4.8 \times 1.00 = 4.8 \text{Hz} \]


B.2.4 Assessment

Based on the above calculated modal properties the floor is classified as class D (Figure 6). The expected OS-RMS$_{90}$ value is approximately 3.2 mm/s.

According to Table 2, class D is suitable for office buildings, i.e. the requirements are fulfilled.

\[
\delta_{\text{max}} = \delta_{\text{slab}} + \delta_{\text{beam}} \\
\delta_{\text{slab}} = \frac{5 \times 4.3 \times 10^{-3} \times 2500^4}{384 \times 34100 \times 2.0355 \times 10^5} = 0.3 \text{mm} \\
\delta_{\text{beam}} = \frac{5 \times 11.97 \times 1500^4}{384 \times 210000 \times 270089 \times 10^4} = 13.9 \text{mm} \\
\delta_{\text{max}} = 0.3 + 13.9 = 14.2 \text{mm} \\
\Rightarrow f_1 = \frac{18}{\sqrt{14.2}} = 4.8 \text{Hz}
\]

Modal Mass

The determination of the Eigenfrequency shows that the load bearing behaviour of the floor can be approximated by a simple beam model. Thus this model is taken for the determination of the modal mass:

\[
M_{\text{mod}} = 0.5 \mu l = 0.5 \times 1220 \times 15 = 9150 \text{ kg}
\]

Damping

The damping ratio of the steel-concrete slab with false floor is determined according to Table 1:

\[
D = D_1 + D_2 + D_3 = 1 + 1 + 1 = 3\%
\]

With

\[
D_1 = 1.0 \text{ (steel-concrete slab)} \\
D_2 = 1.0 \text{ (open plan office)} \\
D_3 = 1.0 \text{ (ceiling under floor)}
\]
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References


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